

WYMAN (J. H.)

Dr. O. W. Holmes

NOTES

with the regards of

J. Wyman

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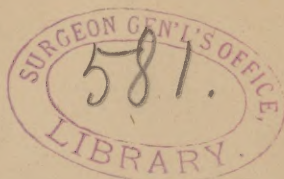
THE CELLS OF THE BEE.

BY

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NOTES ON THE CELLS OF THE BEE.

It is more than a century and a half since Maraldi studied the form of the cells of the hive bee, and described them as hexagonal prisms with trihedral bases, each face of the base being a rhomb, the greater angles of which were $109^{\circ} 28'$, and the lesser $70^{\circ} 32'$.^{*} Twenty-five years later, Reaumur, the most admirable of the observers of insect life, with the view of ascertaining how far such a form was an economical one, proposed to Koenig the following problem,—“Of all hexagonal cells, having a pyramidal base composed of three equal and similar rhombs, to determine that which can be constructed with the least amount of material.”[†] It is a part of the history of this subject, that Koenig's results differed from those of Maraldi by two minutes in each of the angles, the former having made them $109^{\circ} 26'$ and $70^{\circ} 34'$. It has recently been stated that the table of logarithms used by Koenig had an error which would exactly account for the difference.

Admitting an error of two minutes in each of the angles, still the close correspondence between the results of Koenig and the measurements of Maraldi was well fitted to excite the wonder and admiration of all, and from that time to this the belief has prevailed, that the instinct of the bee enables it to construct such a cell as that sought in Reaumur's problem, if not in all cases, at least in the larger portion of them, without sensible error. It were unjust to keep out of sight the fact, that, however correct the measurements of Maraldi may have been, he has left no record of his method of making them, and furthermore, the possibility of measuring the angles of such a structure as the cell of the bee, without liability to an error of one or two degrees in each angle, is denied by competent authorities, since the angles of the cell are nowhere sharply defined and the surfaces are not strictly planes.[‡]

^{*} *Mem. Acad. des Sciences*, 1712.

[†] *Memoires pour servir à l'Histoire des Insectes*, Tom. V. p. 389. Paris, 1740.

[‡] The first person who appears to have called Maraldi's measurements in ques-

The mineralogist, treating the cell as a crystalline form, would not expect a closer approximation to exact measurement than that just stated.

Lord Brougham, who, of later writers, has written the most elaborately on the subject, in his essay entitled *Observations, Demonstrations, and Experiments upon the Structure of the Cells of Bees*, after having himself solved Reaumur's problem, after having obtained solutions of it through others, and after having himself measured the cells, asserts positively that they are constructed in accordance with the form deduced from calculation, and are therefore exact. Having compared the sides of the cell by measurement with a micrometer, he says, "I certainly can find no inequality."* Again, "She [the bee] works so that the rhomboidal plate may have one particular diameter and no other, always the same length, and that its four angles may be always the same";† and he still further adds, "The construction of the cell, then, is demonstrated to be such that no other which could be conceived would take so little material and labor to afford the same room."‡

We have looked carefully through Lord Brougham's essay, for a recognition of the existence of irregularities in the cells, but have found none, except of such as are of microscopic size. "The lines," he says, "may not be exactly even which the bee forms; the surfaces may have inequalities to the bee's eye, though to our sight they seem plane; and the angles, instead of being pointed, may be blunt or roundish, but the proportions are the same: the equality of the sides is maintained, and the angles are of the same size, that is, the inclination of the planes is just. . . . Now, then, the bee places a plane in such a position, whatever be the roughness of the surface, that its inclination to another plane is the true one required."§

Lord Brougham's answer to L'Houillier's criticisms may be cited to the same effect. When the latter speaks of the conditions re-

tion was Father Boscovich, "who had supposed that the admeasurement of the angles was too nice to be accurately performed, and that the coincidence of M. Maraldi's measurements with theory could only arise from his assuming that the angle of inclination of the rhomboidal plane was the same with that of the hexagon, viz. 120° , from which, no doubt, it would follow that the angles of the rhombuses should be $109^\circ 28'$ and $70^\circ 32'$ respectively."—Lord Brougham, *Nat. Theol.*, p. 351.

* *Natural Theology*, London, 1856, p. 224.

† *Ibid.*, p. 197.

‡ *Ibid.*, p. 324.

§ *Ibid.*, p. 191.

quired being such as theory and observation "nearly agree" in giving to the cells, Lord Brougham replies: "The 'nearly' is quite incorrect: there is an absolute and perfect agreement between theory and observation."*

Mathematicians appear to be of one accord in this; viz. if economy of space and wax is sought, that the form of the cell should be the one alleged to have been ascertained by Maraldi, and which was really calculated by Koenig, and by hundreds of others since his time. Careful observations, however, tend to prove that such a cell is rarely, perhaps never, realized. For, while the deviations from the true form do not exceed a certain limit, a piece of comb, ten cells square, can hardly be found in which one or more irregularities do not occur, of such magnitude, that, however they may look to the bee's eye, can be readily detected by man's. The best observers, such as Reaumur, Hunter, the Hubers, and others, have noticed some of these, but as their investigations had for their chief object the clearing up of other points relating to the habits of the bee, the irregularities of the cells were passed by, for the most part, with merely a mention.

Worker Cells.— These will be treated of first, because they are the most numerous. The drones of a hive only amount at the most to a few hundreds, while the workers are estimated at many thousands, and the number of cells is proportional to the number of young reared. All the varieties found in the worker are repeated in the drone and honey cells, though in the last-mentioned kind the variations are the most marked, and some are introduced which are not found in either of the others.

The average *diameter* of a worker cell, measured on a line perpendicular to its sides, as deduced from the following table, is 0.201, or one fifth of an inch, but it may be increased or diminished in different parts of the same comb.† Reaumur expresses his belief that this was the case, but he gives no measurements. The table given below is the result of the examination of four pieces of comb, which were in all re-

* *Op. cit.*, p. 350.

† Reaumur found that twenty worker cells measured four inches less half of a line; "neglecting the half of a line, the diameter of a single cell would be 2.4 lines" (French); and Huber gives the same dimensions, as also Kirby and Spence, who quote their description of everything relating to the bee from Reaumur and Huber. Latreille found that 76 millimeters comprised 14 cells, when measured in one direction, and 14.5 in another.

spects good average specimens. First, a line of ten cells,* arranged in the direction of the diameter, perpendicular to one of the sides, and then two other sets of the same number, similarly arranged in the direction of the other two diameters, and crossing the first, were carefully measured. Three series of such measurements were made from different parts of each comb. The columns marked I., II., III. give the measurements in the direction of the three diameters.

Combs.	I.	II.	III.	Greatest Difference.
	Inch.	Inch.	Inch.	Inch.
A Series, 1	2.04	1.95	1.98	0.09
2	2.04	1.93	1.95	0.11
3	2.10	2.02	1.92	0.18
B Series, 1	2.00	2.03	2.05	0.05
2	1.98	2.02	2.05	0.07
3	2.04	2.05	2.05	0.01
C Series, 1	2.05	2.05	1.98	0.07
2	2.08	2.08	1.98	0.10
3	2.09	2.08	1.98	0.11
D Series, 1	1.93	1.97	1.95	0.04
2	1.97	2.06	1.85	0.21
3	2.00	1.99	2.10	0.11

The greatest aggregate diameter of any one series of ten cells was 2.10 inches, and the least 1.85 inches, making a difference of 0.25 inch, or the diameter of a cell and a quarter. The average difference is, however, a little less than 0.10 inch. These irregularities do not accumulate beyond a certain amount, and those of one portion are often counteracted in another portion of the same row. In a large piece of comb, sixty cells occupied the space of one foot, which would make the diameter of a cell equal to 0.20 inch; nevertheless, ten cells taken from either end, and ten taken from the middle of this same comb, when compared, gave marked differences. This correction is not, however, a constant condition, for we have, perhaps in most instances, found Hunter's statement correct; viz. that the cells gradually increase in size, the last formed being the largest.†

* Ten cells were measured, in order to avoid the accumulating error resulting from the measurement of a series of single cells. The error in the measurement of ten cells is no greater than that of measuring one, and divided among the ten becomes inappreciable.

† Works of John Hunter, Palmer's edition, Vol. IV. p. 436.

It may be asked, if the comb was not built with all its diameters equal, but afterwards accidentally disturbed. The comb is suspended mostly from the uppermost portion, the lowermost hanging free until considerable progress is made, when it is more or less attached by the sides; taking into consideration the material of it, and the weight, when filled with honey, or covered with crowds of bees, it seems quite probable that in a hot day the softened wax would be stretched by its own weight, thus making the transverse diameter of the cells shorter, and the others proportionally longer. To test this, cells from six different pieces of comb were measured in the direction of their three diameters; the result was, that the aggregate transverse diameters of 570 cells was 38.94 inches, and that of the other two was 38.84 and 38.90 inches, respectively. The transverse diameter, the one liable to be shortened, was absolutely a little the longest.

A variation in the diameters does not necessarily bring with it an inequality in the breadth of the sides, or a difference in the angles. If, however, one of the sides is wider or narrower than the others, which it often is, the angle which it makes with the adjoining ones must be greater or less than 120° , the normal angle. In order to be able to measure the sides as accurately as possible, cross sections were made midway between the mouth and base of the cell, where they are thinnest and the angles sharpest. These sections were obtained by filling the cells with plaster of Paris, and after this had hardened, cutting them down to the required point. In this way, all distortion was prevented. The following table gives the result of the measurement of the different sides of a series of twelve cells.

Sides.	CELLS.											
	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.
1	In. 0.14	In. 0.14	In. 0.14	In. 0.13	In. 0.11	In. 0.10	In. 0.12	In. 0.07	In. 0.14	In. 0.00	In. 0.13	In. 0.10
2	0.12	0.12	0.14	0.11	0.13	0.14	0.14	0.15	0.13	0.14	0.15	0.14
3	0.15	0.12	0.11	0.14	0.13	0.14	0.13	0.15	0.09	0.14	0.13	0.14
4	0.11	0.13	0.14	0.10	0.11	0.11	0.12	0.11	0.14	0.10	0.13	0.12
5	0.13	0.12	0.14	0.12	0.13	0.12	0.14	0.11	0.10	0.14	0.09	0.13
6	0.14	0.13	0.12	0.14	0.13	0.11	0.14	0.14	0.09	0.14	0.10	0.15

Smallest side in 72 cells, 0.070 inch.

Average " " " " 0.125 "

Longest " " " " 0.150 "

Of all the parts of the cell there is none where the variation is more striking than in the *rhombic faces* of the base. This fact is the more noteworthy, since it is upon these, and the angles they make with each other and the sides, that rests the nicest part of the problem relating to the adaptation of the cell to the contained bee. The relative size of the faces may be so changed that two of them make nearly the whole of the base, while the third almost vanishes, or one of the faces may have any size between this extreme and the normal one.

The fourth face, which has been so often noticed, has generally been spoken of as belonging more especially to those cells which are intermediate between the cells of drones and workers. Although it occurs in these, we have found it quite common in the middle of pieces of comb consisting solely of either worker, drone, or honey cells.* In one piece of worker comb containing about five hundred cells, nearly all had a fourth face.

The causes which lead to the introduction of the fourth face are chiefly two, — irregularity in the size of the cells, and incorrect alignment of them on the two sides of the comb. Each cell on one side of the comb being normally in contact, by its rhombic faces, with three cells on the other, and these fitting exactly, if a cell is increased, it will project beyond them, and thus come in contact with a fourth, and a new face will be formed. We have seen this happen in a single cell, but very commonly a row of cells increases for four or five cells, and gradually diminishes again to the ordinary size. With this increase and decrease of the cell, the fourth face comes and goes.

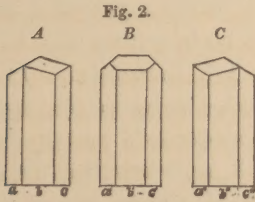
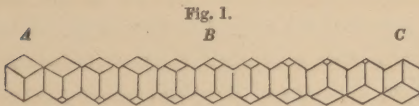
Incorrect alignment is the more common cause.† If a given row of

* These were studied either after cutting away the body of the cells, leaving only the basal plate which separates those of opposite sides, or by means of casts obtained by filling the cells with plaster of Paris. After this last has dried, if the mass is heated, the wax is absorbed by the plaster, when the casts of the two sets of cells separate. In old brood-combs, where many successive cocoons have been spun, these form a thick and resisting cast of the base of the cell and may be extracted, giving its precise form. In some instances, fourteen distinct layers of cocoons were counted, showing the number of broods which had occupied the cells.

† This introduction of the fourth face to the basal pyramid, through incorrect alignment, was thoroughly investigated several years since by Mr. Chauncey Wright, of the Nautical Almanac Office, and who, at the same time, constructed models illustrating his views. These models are deposited in the Museum of Comparative Anatomy and Physiology at Cambridge. For a discussion of various points connected with the geometry of the cell, see his article, entitled *The Economy and Symmetry of the Honey-Bee's Cell*, in the *Mathematical Monthly* for June, 1860.

cells on one side of the comb ceases to be parallel with those on the other, with which it was connected when the comb was begun, and diverges from them, it is gradually transferred to a new series; as the cells come in contact with those of the new series, the fourth side appears, and, at the same time, one of the original sides, viz. that directly opposite to it, is gradually diminished, and may vanish. This divergence is, however, sometimes insufficient to make the separation of the rows complete, and may gradually diminish again, as they are extended by the construction of new cells, so as to bring them back to the original position, when the irregularity is corrected.

If, however, the separation of the two rows at length becomes complete, so that one of the faces is lost, and a new one formed, all the basal portion of the cell becomes reversed, as will be seen by reference to



Figs. 1 and 2; the first representing the cells when the base is viewed, and the second when looked at perpendicularly to one of the sides. In both figures *A* indicates the ordinary form of the cell. The whole series of Fig. 1 shows the gradual introduction of the new face, which is seen on the lower border, and the

elimination of one of the original faces, which is seen on the upper border. At *B*, which is intermediate between the two extremes, the four faces consist of two equal rhombs, — one of which is the outgoing and the other the incoming one, — and two equal hexagons. *B*, Fig. 2, represents the sides of the same cell, which, instead of forming three trapeziums, as at *A*, *a*, *b*, *c*, now form two pentagons, *a'* and *c'*, and a parallelogram, *b'*. At *C*, Figs. 1 and 2, the forms are in all respects the reverse of those of *A*. *A* and *C* are symmetrical with each other, and *B* is symmetrical in itself. No precise number of cells is necessary for the purpose of making this transition, for it may take place in two or three, or extend through a long series, as in Fig. 1.

There is another variation which we have noticed twice, — once in drone, and once in worker comb, involving a large number of cells. If a piece of normal comb be held in the position in which it was built,

two of the opposite angles of the hexagon, Fig. 3, *A*, *a*, will be in the same vertical line, and two of the sides will be parallel to this. The same is true of the opposite side of the comb; and thus all the corresponding parts of the cells on the two sides will be parallel. In the deviation we are now noticing, the change is like that represented in *A*, where the cell *a* is in its true position, while the cell *b*, which is from the opposite side, and is in contact with *a*, varies from it by

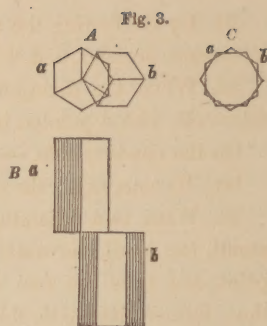
about 30° . If we look at these two cells in the direction of their sides as at *B*, the prism *a* will have one of its angles towards the eye, and *b* one of its sides. If rows of cells are constructed on each of the sides *a* and *b*, Fig. 3, *B*, it will be seen that the rows thus formed on the two faces of the comb will cross each other continually. A modification of this variety is seen at *C*, where the axes of the two adjoining prisms, instead of being separated as usual by the semidiameter of a cell, coincide; consequently, as the apices of the angles of *a* project beyond the sides of *b*, *a* will not only be in contact with *b*, but by its angles with the six cells by which *b* is surrounded. In either of these cases the pyramidal base becomes impracticable, and the flat bottom of the cell is substituted for it almost as a matter of necessity. The bottoms of the cells being flat, it is obvious that the change of position by rotation of the cell on its axis may be carried to any extent, without leading to an interference with the cells of the opposite side; in fact, several degrees of it have been observed.

Since the mouths of such cells are in the same plane with those normally constructed in the same comb, and since the pyramidal base is cut off, they are shortened by an amount equal to the height of that of the base, and therefore are of a proportionately less capacity than the normal cell. Nevertheless, such truncated cells are used for rearing the young, and, like the others, were found to contain cocoons.

In curved or bent combs the cells on the concave side tend to become narrower, while those on the other tend to become broader towards their mouths. The bees meet this emergency in one of the following ways:—

On the convex side,—

1st. By allowing the cavity of this cell to become broader, without any correction being made.



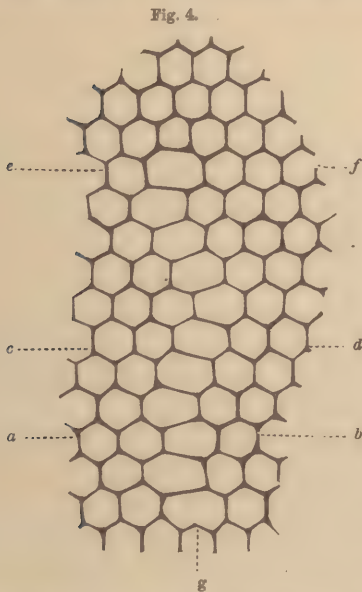
2d. By thickening the walls of the cell in proportion to its tendency to become broader, and thus keeping the diameter of its cavity uniform.

3d. When the divergence reaches a certain amount, by making a false-cell, with a pointed bottom, between the diverging cells.

On the concave side, —

1st. By narrowing the cell towards its mouth.

2d. When two adjoining cells converge so much as to render the mouth too small, the walls between them are suppressed at a certain point, and thus the two mouths are merged, and the compound cell thus formed has a double base, and but one entrance, the two cells being combined, as are certain kinds of twin crystals, or of double monsters. The form of the mouth under these circumstances is, however, liable to a considerable range of variation, as in the central line



of cells in Fig. 4,* in which are a variety of hexagons. That on the line *a, b* has three sides at one end, united by two long sides with one at the other, and thus two of the opposite sides are not parallel; at *c, d*, two sides at either end are united by two long sides, these last being parallel; and at *e, f*, the mouth of the compound cell has seven sides. Each has a partition at its base, separating the two originally distinct cells, and each was lined with a cocoon, showing that it had been used for rearing young.

In combining the mouths of two adjoining cells, it will be seen that this does not consist merely in suppressing the partition between

them; for if this were so, each of the long sides would contain more or less of an angle, as at the lower side of *g*, according to the degree of convergence, until three of the sides of each of the combining cells had disappeared. Instead of this, the portions of two sides forming the angle just referred to are replaced by one straight side, as on the upper side of *g*, and in both of the long sides of the undulating line of cells above it.

* Figs. 4, 5, and 6 are made from impressions obtained directly from the comb and transferred to wood. They represent the forms of the cells exactly.

Drone Cells. — These are liable to substantially the same variations as the worker cells. Reaumer observed that they were larger by one ninth in one diameter than in another.* Four pieces of drone comb gave the following measurements.

Combs.	I.	II.	III.	Greatest Difference.
	Inch.	Inch.	Inch.	Inch.
A Series, 1	2.63	2.72	2.67	0.09
2	2.70	2.60	2.72	0.12
3	2.80	2.58	2.60	0.20
B Series, 1	2.47	2.70	2.54	0.23
2	2.54	2.50	2.55	0.05
3	2.56	2.58	2.37	0.21
C Series, 1	2.54	2.55	2.47	0.08
2	2.59	2.50	2.55	0.09
3	2.64	2.61	2.68	0.07
D Series, 1	2.40	2.47	2.46	0.07
2	2.45	2.43	2.36	0.09
3	2.67	2.52	2.49	0.18

I., II., III., in the above table, indicate the diameters drawn perpendicularly to the three pairs of sides of the hexagons, and series 1, 2, 3 indicate measurements of cells made from three portions of each comb. Ten cells were measured in each case.

In comparing all of the above measurements, it is found that the smallest aggregate diameter of any ten cells is 2.36 inches, and the largest, 2.80 inches, making an extreme difference of 0.44 inch, or the diameter of a drone and almost that of a worker cell in addition. The greatest variation in any one series was 0.21, or a little more than four fifths of the diameter of a drone cell, which is somewhat less than the quantity given by Reaumer.

The following measurements from twelve consecutive rows of cells, of ten each, from the middle of a piece of drone comb, show the progressive variation from one row to another.

1st row	2.47 inches.	7th row	2.64 inches.
2d "	2.50 "	8th "	2.67 "
3d "	2.51 "	9th "	2.67 "
4th "	2.54 "	10th "	2.66 "
5th "	2.58 "	11th "	2.63 "
6th "	2.62 "	12th "	2.65 "

* *Op. cit.*, Tom. V. p. 398.

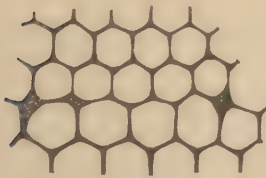
Transition Cells. — As drone are one fifth larger than worker cells, and as both are combined in one and the same piece of comb, a transition cannot be made from one to the other without some disturbance in the regularity of the structure. It would be a nice problem to determine the way in which this could be effected with the greatest economy of space and material. The bees do not appear to have any systematic method of making such a change. More commonly, they effect it by a gradual alteration of the diameters, thus enlarging a worker into a drone, or narrowing a drone into a worker cell. This alteration is usually made in from four to six rows. The following table gives an illustration of the rate of alteration in such a case, beginning with four drone cells of the usual size, and ending with four worker cells.

Four drone cells measured in the

1st row	1.02 inch.
2d	"	.	.	.	0.97 "
3d	"	.	.	.	0.95 "
4th	"	.	.	.	0.86 "
5th	"	.	.	.	0.83 "
6th	"	.	.	.	0.80 "

This last measurement exactly equals that of four worker cells. The rate of the reduction of the size of the cell is not uniform, the differences between successive rows being .05, .02, .01, .03, .03 inch. We have, however, seen the transition made with two rows of transitional

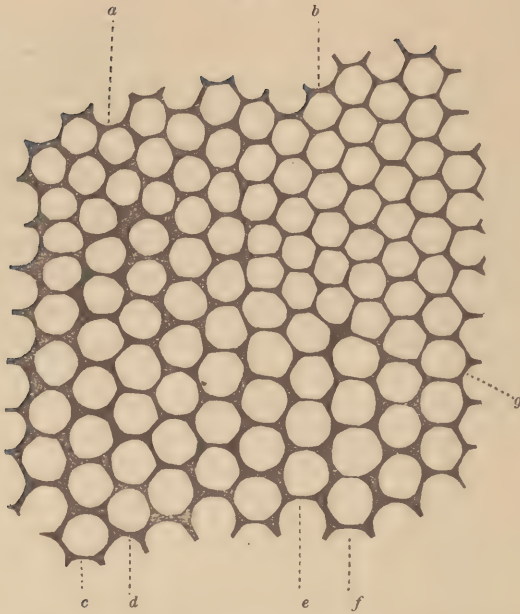
Fig. 5.



cells, and as in Fig. 5, with only one. In this last case, the regularity of two adjoining rows is sacrificed.

In consequence of the gradual narrowing or widening of the transition cells, the comb tends to become more or less triangular and the cells to become disturbed. The bees counteract this tendency by the occasional intercalation of an additional row, of which two instances are given in Fig. 6, at *a* and *b*, where three rows of worker cells are continuous with two of drone cells, *c d* and *e f*; or, reversing the statement, and supposing the transition, as in the building of the comb, is from worker to drone cells, a row of the latter is from time to time omitted as the rows *a* and *b*; in this way,

Fig. 6.



the regularity of the comb is preserved.* This, however, is not done at definite intervals; for in one piece of comb two intercalated series were nine cells apart, in another, six, and in another, four.

Mr. Langstroth has given a good figure, illustrating the form of the mouths of some of the cells where the worker and drone cells come together.†

The presence of a fourth face in the base of the transitional cells is by no means constant, as asserted by several observers, for we have seen the change from worker to drone cells without the fourth face appearing in any of them.

In all the transitional cells of brood-comb cocoons are invariably found, showing that they have been occupied. It is obvious that some of these would be either too large for a worker or too small for a

* This figure was made from a piece of old brood-comb, in which the lip of the drone cells was very much thickened, and the mouths were almost circular. There is nothing abnormal in this, except at those points where the row of intercalated cells, as *a* and *b*, connect with the drone cells.

† Treatise on the Bee, p. 74 and Pl. XV.

drone. It would, therefore, be of considerable interest to know whether such cells are occupied by one or the other of these kinds of bees. The determination of this point is important on another account. Siebold has ascertained that drones do not require impregnation, while the workers as well as the queens do; and as the act of impregnation is voluntary with the queen, she is supposed to have some guide to inform her whether a given egg is to become one or the other kind, for she never makes a mistake and impregnates an egg in a drone cell, or omits to impregnate one in a worker cell. Siebold, therefore, supposes that the queen is guided by the size of the mouth of the cell, and if the abdomen touches one kind, impregnation takes place, and if the other, not. The transitional cells being intermediate, would not by their size give her the usual indication.

Honey Cells. — When the stock of honey becomes greater than the ordinary brood cells will contain, the bees either enlarge these, or add to them other cells often of larger capacity, or construct a new comb, devoted entirely to the storing of honey. While the cells of this last are built unequivocally in accordance with the hexagonal type, they exhibit a range of variation from it which almost defies description. Of all who have written on the subject, Mr. Langstroth is the only one we have met with who seems to have particularly mentioned their irregularity, which he does in the following words: "Those [cells] in which the honey is stored vary greatly in depth, while in diameter they are of all sizes, from that of a worker to that of drone cells."* We have found them even 2.10 inches in depth, or four times that of a worker cell; sometimes they are square or pentagonal; their alignment is rarely if ever exact, so that the presence of a fourth face is more common than with the other kinds. The basal pyramid changes constantly; the cast of a piece of comb, containing over four hundred cells, showed but few in which there was not some irregularity obvious to the eye; either the faces were unequal, or there was a fourth, and even a fifth face, or the pyramid was too high or too low, or suppressed, or the body of the cell was not equilateral, or its angles too large or too small. The normal angle which one side makes with its adjoining ones is 120° ; the following measurements, taken from casts of average specimens, exhibit a degree of variation by no means unusual.

* On the Honey-Bee, p. 74. New York, 1859.

Angles.	CELLS.			
	I.	II.	III.	IV.
1	117°	117°	112°	113°
2	122	124	127	130
3	121	116	120	122
4	110	119	114	110
5	135	125	125	126
6	115	118	121	117

Largest angle, 135°.0

Average of the 24 angles, 119°.5

Smallest, 110°.0

The above measurements were made with an accurate goniometer; those of cells I. and II. by Professor Cooke, and of III. and IV. by the author, and each is the average of three; but, in nearly every case, there is an error of from one to three degrees, which is inseparable from the measurement of surfaces and angles which are not exact.

When honey cells are built on a curved dividing wall, the bees seem to make no attempt to correct the effect of the converging and diverging lines. In the brood-combs they usually make an attempt, at least, to keep the cavity of the cells of the usual shape; but in the honeycomb we have seen the mouths of the cell in one diameter expanded to double their usual size. The most of the irregularities seem to have no obvious cause, but all looks as if the bees, aware that close conformity to the type-form was unnecessary for the mere storing of honey, became careless in executing their work.

The distribution of the wax in the sides and angles of the cells, as seen in the sections, appears to the naked eye quite regular; but, with the aid of a low power, is often quite the reverse. One can easily detect an inequality in the thickness of the walls, — two different walls of the same cell, or two parts of one and the same wall being not unfrequently the one double the thickness of the other. The excavation of the angles, though sometimes wonderfully exact, is frequently done in such a way that the apices of opposite angles do not correspond. This is equally true of all of the three kinds of cells. In the cells, and still better in the casts of them, one can easily observe the fact that the edges of the sides are never exactly planes, and that consequently the line of union of two adjoining sides is somewhat undulating.

The statements made in the foregoing pages tend to show that the

cell of the bee has not the strict conformity to geometrical accuracy so often claimed for it, but, as the best observers have maintained, is liable to marked variations, chief among which the following may be mentioned.

1st. The diameters of worker cells may so vary, that ten of them may have an aggregate deviation from the normal quantity equal to the diameter of a cell. The average variation is a little less than one half that amount, viz. nearly 0.10 inch, in the same number of cells.

2d. The width of the sides varies, and this generally involves a variation of the angles which adjoining sides make with each other, since the sides vary not only in length, but in direction.

3d. The variation in the diameters does not depend upon accidental distortion, but upon the manner in which the cell was built.

4th. The relative size of the rhombic faces of the pyramidal base is liable to frequent variation, and this where the cells are not transitional from one kind to another.

5th. When a fourth side exists in the basal pyramid, it may be in consequence of irregularity in the size of the cells, or of incorrect alignment of them on the two sides of the comb.

6th. Ordinarily, the error of alignment does not amount to more than one or two diameters of a cell. But occasionally the rows of cells on one side of the comb may deviate from their true direction with regard to those on the other, to the extent of 30° . In consequence of this deviation and the continual crossing of the rows on opposite sides, the pyramidal base is not made, and the cell is thereby shortened.

7th. When a piece of comb is strongly curved, or two portions form an angle with each other, there is no constant way in which the tendency to the distortion of the cells is met; consequently the cells found at the curves or angles have a variety of patterns.

8th. Deformed worker and drone cells are used for rearing the young.

All of these variations are found in the three different kinds of cells, but are much more frequent and marked in the honey than in either worker or drone cells. In view of the frequency of such, however near the bee may come to a typical cell in the construction of its comb, it may be reasonably doubted whether a type cell is ever made. Here, as is so often the case elsewhere in nature, the type-form is an ideal one, and, with this, real forms seldom or never coincide. Even in crys-

tallography, where the forms are essentially geometrical, we are told that "natural crystals are always more or less distorted or imperfect," and that "the science of crystallography could never have been developed from observation alone";* i. e. without recourse to ideal conceptions. An assertion, like that of Lord Brougham, that there is in the cell of the bee "perfect agreement" between theory and observation, in view of the analogies of nature, is far more likely to be wrong than right; and his assertion in the case before us is certainly wrong. Much error would have been avoided, if those who have discussed the structure of the bee's cell had adopted the plan followed by Mr. Darwin, and studied the habits of the cell-making insects comparatively, beginning with the cells of the humble-bee, following with those of the wasps and hornets, then with those of the Mexican bees (*Melipona*), and, finally, with those of the common hive-bee. In this way, while they would have found that there is a constant approach to the perfect form, they would at the same time have been prepared for the fact, that even in the cell of the hive-bee perfection is not reached. The isolated study of anything in natural history is a fruitful source of error.

Since bees give so much variety to the forms of their cells, and can adapt them to peculiar circumstances, some of which do not occur in nature, as, for example, in Huber's experiment with the glass surface, which last they so persistently avoided, and in view of the fact, too, that in meeting a given emergency they do not always adopt the same method, one is driven to the conclusion that the instinct of one and the same species either is not uniform in its action and is quite adaptive in its quality, or to admit, with Reaumer, that bees work with a certain degree of intelligence.

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* Professor Cooke, Religion and Chemistry, p. 287.

